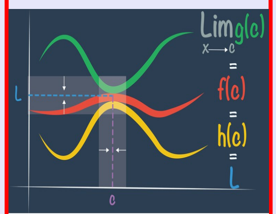
Math 261

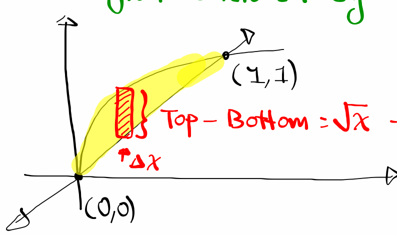
Fall 2022

Lecture 46



Feb 19-8:47 AM

1) Draw the region enclosed by $y=\sqrt{x}$, and $y=x$.



2) Find its area. $\int_0^1 (\sqrt{x} - x)\,dx = \int_0^1 [x^{1/2} - x]\,dx$

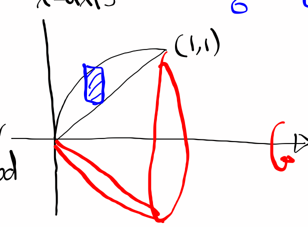
$= \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{2} - 0$

$= \frac{4-3}{6} = \boxed{\frac{1}{6}}$

3) Rotate this region about x-axis and find its volume.

1) Ref. Rec. ⊥ A.O.R.
2) Region is not 100% attached to A.O.R.

} Washer Method

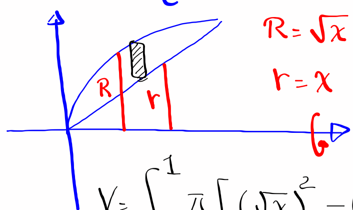


Nov 21-8:46 AM

How to set-up the washer method:

$$V = \int_a^b \pi\left[R^2 - r^2\right]dx \quad \text{when}$$

$$V = \int_c^d \pi\left[R^2 - r^2\right]dy \quad \text{when}$$



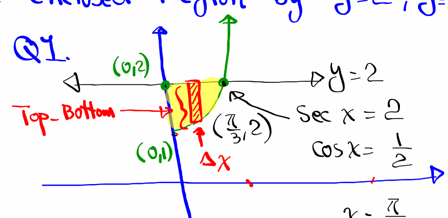
$$V = \int_0^1 \pi\left[(\sqrt{x})^2 - (x)^2\right]dx$$

$$= \pi\int_0^1 (x - x^2)\,dx = \pi\left[\frac{x^2}{2} - \frac{x^3}{3}\right]\Big|_0^1 = \pi\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \boxed{\frac{\pi}{6}}$$

Nov 21-8:54 AM

Draw the enclosed region by $y=2$, $y=\sec x$, $x=0$, in QI.



$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

Find its area.

$$A = \int_0^{\pi/3} \left[2 - \sec x\right]dx$$

$$= \left[2x - \underbrace{\ln|\sec x + \tan x|}_{\text{Calc 2}}\right]\Big|_0^{\pi/3}$$

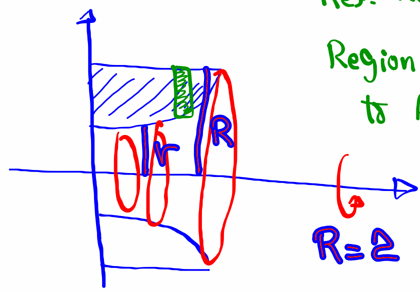
$$= \left(2\cdot\frac{\pi}{3} - \ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right|\right) - \left(2\cdot 0 - \ln|\sec 0 + \tan 0|\right)$$

$$= \frac{2\pi}{3} - \ln|2 + \sqrt{3}| - 0 + 0$$

$$= \boxed{\frac{2\pi}{3} - \ln(2+\sqrt{3})}$$

Nov 21-8:59 AM

Rotate this region by x-axis, and find its Volume.



Ref. Rect. ⊥ AOR

Region is not 100% attached to A.O.R

washer method.

$R=2$, $r=\sec x$

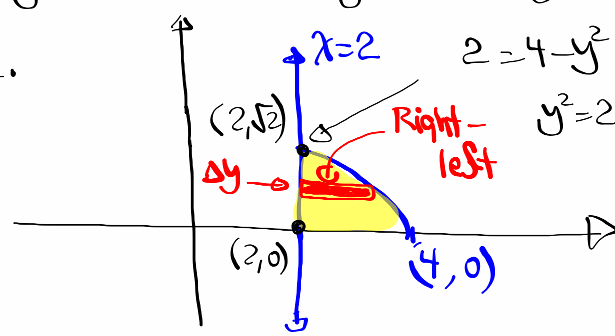
$$V=\int_0^{\pi/3}\pi\left[2^2-\sec^2 x\right]dx=\pi\left[4x-\tan x\right]\Big|_0^{\pi/3}$$

$$=\pi\left[\frac{4\pi}{3}-\tan\frac{\pi}{3}-0\right]$$

$$=\boxed{\pi\left[\frac{4\pi}{3}-\sqrt{3}\right]}$$

Nov 21-9:10 AM

Draw the region enclosed by $x=4-y^2$ and $x=2$ in QI.



$2=4-y^2$

$y^2=2$

$y=\sqrt{2}$

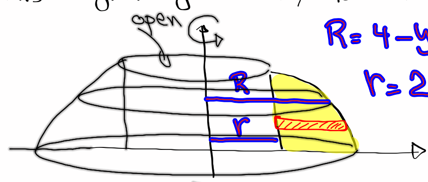
$$A=\int_0^{\sqrt{2}}\left[\text{Right}-\text{left}\right]dy=\int_0^{\sqrt{2}}\left[4-y^2-2\right]dy$$

$$=\int_0^{\sqrt{2}}(2-y^2)dy=\left(2y-\frac{y^3}{3}\right)\Big|_0^{\sqrt{2}}$$

$$=2\sqrt{2}-\frac{2\sqrt{2}}{3}=\boxed{\frac{4\sqrt{2}}{3}}$$

Nov 21-9:16 AM

Rotate this region by Y-axis, and find its Volume



$R = 4 - y^2$

$r = 2$

Washer method $\int_0^{\sqrt{2}} \pi \left[(4-y^2)^2 - 2^2 \right] dy$

$= \pi \int_0^{\sqrt{2}} \left[16 - 8y^2 + y^4 - 4 \right] dy$

$= \pi \left[12y - \frac{8y^3}{3} + \frac{y^5}{5} \right] \Big|_0^{\sqrt{2}}$

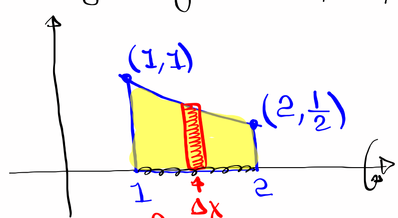
$= \pi \left[12y - \frac{8\, y^2 \cdot y}{3} + \frac{y^2 \cdot y^2 \cdot y}{5} \right] \Big|_0^{\sqrt{2}}$

$= \pi \left[12\sqrt{2} - \frac{16\sqrt{2}}{3} + \frac{4\sqrt{2}}{5} \right]$

$= \pi \left[\frac{180\sqrt{2} - 80\sqrt{2} + 12\sqrt{2}}{15} \right] = \frac{\pi \cdot 112\sqrt{2}}{15}$

Nov 21-9:23 AM

Draw an enclosed region by $x=1$, $x=2$, $y=0$, and $y = \frac{1}{x}$.



Find its area.

$A = \int_1^2 \left[\text{Top} - \text{Bottom} \right] dx$

$= \int_1^2 \left(\frac{1}{x} - 0 \right) dx = \underbrace{\int_1^2 \frac{1}{x} dx}_{\text{Calc 2}}$

$= \ln x \Big|_1^2$

$= \ln 2 - \ln 1 = \boxed{\ln 2}$

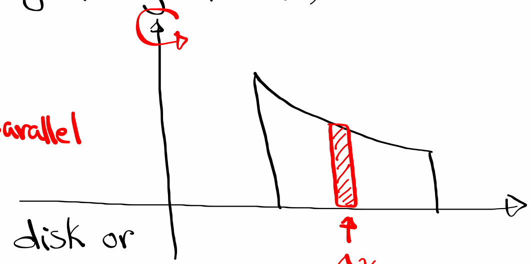
Rotate by x-axis, find Volume. Disk

$V = \int_1^2 \pi \left[\left(\frac{1}{x} \right)^2 \right] dx = \pi \int_1^2 x^{-2} dx = \pi \cdot \frac{x^{-1}}{-1} \Big|_1^2 = -\pi \cdot \frac{1}{x} \Big|_1^2$

$= -\pi \left[\frac{1}{2} - 1 \right] = \boxed{\frac{\pi}{2}}$

Nov 21-9:30 AM

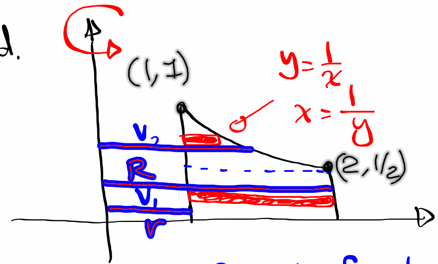
Rotate the region by Y-axis, and find its Volume.



Ref. Rec. is parallel to A.O.R.

We cannot use disk or washer method. We must use shell method.

I don't know shell Method.



$V_1=\int_0^{1/2}\pi\left[2^2-1^2\right]dy$

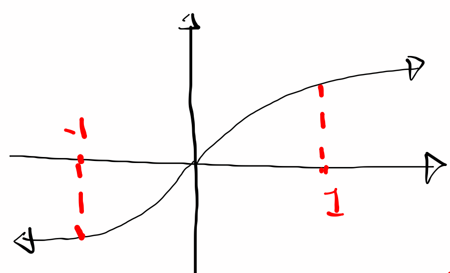
$V_2=\int_{1/2}^{1}\pi\left[\left(\frac{1}{y}\right)^2-1^2\right]dy$

Make Sure to finish this.

$V=V_1+V_2$

Nov 21-9:39 AM

Find f_{ave} of $f(x)=\sqrt[3]{x}$ on $[-1,1]$.



$f(x)=\sqrt[3]{x}$ odd function

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{1-(-1)}\int_{-1}^{1}\sqrt[3]{x}\,dx$$

$$=\frac{1}{2}\int_{-1}^{1}\sqrt[3]{x}\,dx=\frac{1}{2}(0)=\boxed{0}$$

If $f(x)$ is an odd function

$$\int_{-a}^{a}f(x)\,dx=\boxed{0}$$

Nov 21-9:49 AM